

Pentaquarks in the $\overline{10}_F$ - and 10_F -plets

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We discuss the mass splittings of the pentaquark 10_F -, and $\overline{10}_F$ -plet due to the quark mass differences, as well as the 10_F -plet's median mass relative to that of the $\overline{10}_F$ -plet of pentaquarks for both the flavour-spin and the colour-spin hyperfine interactions, and for both parities of the pentaquark ground states. We show that the colour-spin interaction leads to degenerate 10_F - and $\overline{10}_F$ -plets when the parity is even, and a median mass splitting of 75 MeV for odd parity. The flavour-spin interaction leads to $10_F - \overline{10}_F$ median mass splittings of 200 MeV and 40 MeV, for even and odd parities, respectively. We display mass relations between 10_F - and $\overline{10}_F$ -plets and analyze the presently known baryon resonances in this light.

Introduction The recent wave of experimental activity led to the observation of two purely exotic pentaquark states [1, 2]. These are the strangeness $S = 1$, $\Theta^+(1540)$ and the strangeness $S = -2$, $\Xi^{--}(1862)$ resonances, with very small widths, of one MeV or more, but smaller than 25 MeV and 18 MeV respectively (limits imposed by the experimental resolution of [1] and [2]). Their flavour quantum numbers present indubitable evidence that they consist of, at the very least, four valence quarks and one valence antiquark, i.e., that they are “pentaquarks” ³. The former state has been independently confirmed [3, 4, 5, 6, 7, 8, 9], whereas the latter awaits confirmation, see Ref. [10]. At present their spins and parities are experimentally unknown, but the chiral soliton model (χSM), which so accurately predicted the $\Theta^+(1540)$ mass and width, demands spin-parity $J^P = \frac{1}{2}^+$, both for Θ^+ and Ξ^{--} , as members of the same flavour antidecuplet [11]. Here we shall adopt the same point of view inasmuch as both standard versions of the constituent quark model, *viz.* the flavour-spin hyperfine interaction (HFI) and the colour-spin hyperfine interaction model, predict even parity pentaquarks as their ground states, as shown below. For completeness, however, we shall study both parity cases and shall discuss the relative positions of the corresponding states.

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³Of course, this state may contain higher Fock space components, such as septaquarks, with identical quantum numbers. We apply Occam's razor to such components.

Pentaquarks can be thought of as being constructed from a q^3 (baryon) and a $q\bar{q}$ (meson) subsystem, which means that they can be either in the $8_F \times 8_F$, or in the $10_F \times 8_F$ direct products of $SU(3)$. If the SAPHIR Collaboration results [5] can be taken as conclusive, then the Θ^+ has no isospin partners and thus (making the most conservative assumption) corresponds to top corner of the weight diagram, see Fig. 1, of the $SU_F(3)$ $\overline{10}$ -plet [11]. Note that only the Clebsch-Gordan series of $8_F \times 8_F = 27_F + 10_F + \overline{10}_F + 2(8_F) + 1_F$ contains the $\overline{10}$ -plet while in the direct product decomposition $10_F \times 8_F = 35_F + 27_F + 10_F + 8_F$ the $\overline{10}_F$ does not appear. For this reason, for the time being, we concentrate only on the $8_F \times 8_F$ product. Moreover, the $\Xi^{--}(1862)$, if its detection is confirmed, is likely to be a member of an isoquartet, thus providing one (the left-hand side) bottom corner in the weight diagram of the $SU_F(3)$ $\overline{10}$ -plet.

An immediate challenge is to give a quark model interpretation (quark wave function) of the solitonic $\overline{10}$ -plet states, as the relation between the latter and the quark model is tenuous at best ⁴, and a dynamical explanation of the $\overline{10}$ -plet's low mass, as well as that of the absence of other flavour multiplets. The problem lies in the large number of possible pentaquark $\overline{10}$ -plet states in the quark model, among which one has to find the lowest lying one, as well as in the non-uniqueness of the constituent quark interactions.

In the constituent quark model one usually assumes two parts to the quark-quark interaction: (1) a long range spin-independent part that confines quarks of any spin or flavour alike: with this interaction all pentaquarks are degenerate and (2) a short range spin-dependent hyperfine part that determines the mass splittings between various spin/flavour multiplets, of which there are two “standard” models, as mentioned above. It is this second part of the interaction that ought to lower the even parity $\overline{10}_F$ -plet's mass and keep other states' masses higher.

We wish to study exotic pentaquarks other than those belonging to $\overline{10}_F$ in the context of the constituent quark model(s). The 27_F -plet and the 35_F -plet have the largest numbers of real, or ortho-exotics members. Both multiplets seem to have been eliminated as candidates for the “home” of Θ^+ by the SAPHIR Collaboration results [5], however. ⁵

As the 1_F and the 8_F s are all cryptoexotics, they may mix, or be confused with lower (q^3) Fock components with identical quantum numbers, leaving us with only 10_F . ⁶ In particular we shall concentrate on the $J^P = 1/2^+$ pentaquark 10_F -plet, which does not have a counterpart in the soliton models, where the decuplet can be only in the $J = 3/2$ state, due to the Wess-Zumino term. It is therefore reasonable to ask first where the para-exotic ⁷ members, of the 10_F -plet lie in comparison with the $\overline{10}_F$ -plet in the constituent quark models.

In the absence of a strong hyperfine interaction and of $SU(3)_F$ symmetry breaking all of the $SU(3)_F$ multiplets are degenerate (in this case we have an exact $SU(6)_{FS}$ symmetry). After “turning on” the hyperfine interactions, various $SU(3)_F$ multiplets' energies

⁴Indeed, in one version of the χSM , the Skyrme model, there are no quarks at all.

⁵This does not mean that the 27_F -plet and the 35_F -plet do not exist at some higher mass, (see e. g. Ref. [12]), which fact would make them more difficult to detect experimentally.

⁶Strictly speaking even the 10_F -plet is not a real, or ortho-exotic: the 10_F -plet certainly shows up in the q^3 spectra, but, for even parity usually with spin $3/2$, in the $SU(6)$ (56)-plet. It is only in the $2\hbar\omega$ shell that spin $1/2$, even parity q^3 , 10_F -plets appear, in the $SU(6)$ (56, 2^+) sector but those states are sufficiently heavy, see e. g. [13], so as not to be confused, or mixed with genuine pentaquarks.

⁷Those states that are not pure exotics, but do not mix with octet members.

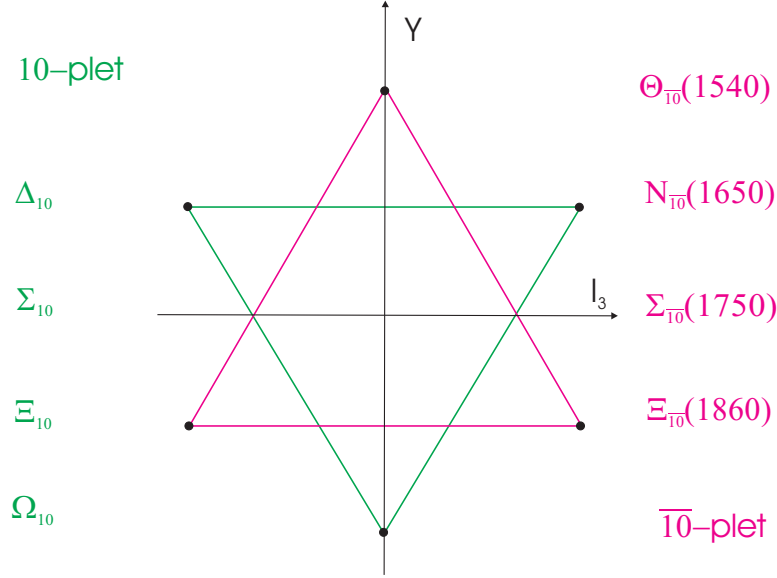


Figure 1. The weight diagrams of the $\overline{10}_F$ - and the 10_F -plets in $SU(3)_F$ in the absence of any hyperfine interactions.

are shifted by different amounts, but the members of individual multiplets remain degenerate. Once the $SU(3)_F$ symmetry is broken, members of various $SU(3)_F$ multiplets start mixing; clearly this mixing depends on both the $SU(3)_F$ symmetry breaking and the (HFI induced) energy splitting of unbroken multiplets, i.e., on the hyperfine interaction (“ $SU(6)_{FS}$ symmetry breaking”).

In order to simplify the subsequent discussion we shall do our analysis in two steps: firstly we shall separately look at the effects of $SU(3)_F$ symmetry breaking on the $SU(3)_F$ multiplets in the “free” quark model and at those of the hyperfine interactions; secondly we shall look at their combined effect. Note that this is not the whole story, as we shall not take into account the effects of $SU(3)$ symmetry breaking upon the HFI itself, which need not be negligible. This, however, is within the spirit of our “schematic model”: Such “higher order” corrections will have to be analysed elsewhere.

To answer these questions we must first find the $SU(3)_F$ symmetry breaking pattern for the pentaquarks in the absence of HFI, and check if it would be possible to fit $\Theta^+(1540)$ and $\Xi^{--}(1862)$ into the $\overline{10}_F$ -plet. Indeed, several authors [14] have already expressed reservations with regard to this option. After calculating the pentaquark 10_F -plet and $\overline{10}_F$ -plet spectrum in the presence of $SU(3)_F$ symmetry breaking, we shall turn on the HFI and see how that affects the whole picture.

Mass formulas for pentaquarks in the absence of HFI

The $SU(3)$ weight diagrams of the $\overline{10}_F$ and the 10_F are depicted in Fig. 1 Each “corner” of the 10_F -plet describes a para-exotic pentaquark state, as in the case of $\overline{10}_F$. We denote these pentaquarks by $\Delta_{10}^{++}(uuuq\bar{q})$, $\Delta_{10}^{--}(dddq\bar{q})$ and $\Omega_{10}^{--}(sss\bar{q}q)$ respectively, whereby the quark labels are to be understood merely as the pentaquarks’ (schematic) quark contents and not as their flavour wave functions.

Due to the large multiplicities of the 8_F -plet and of the 10_F -plets in the pentaquark Clebsch-Gordan series, $3_F \times 3_F \times 3_F \times 3_F \times \bar{3}_F = 35_F + 3(27_F) + 4(10_F) + 2(\bar{10}_F) + 8(8_F) + 3(1_F)$, (8-, and 4-fold, respectively), disentangling the mixings of these two kinds of states seems like a hopeless task at the present moment. We rather concentrate on those members of the decimet that do not mix with the octets. This does not mean that we may forget about the mixings of the para-exotic decimet states among themselves or with states from “higher” multiplets: there are states in the 35_F -plet that do mix with (at least some of) them. Note, however, that not all of these multiplets need appear among the ground states of any given spin and parity, e.g. there is only one octet and no singlets among the $J^P = 1/2^+$ ground state q^3 baryons. By the same token, a smaller number (than four and two respectively) of 10_F -plets and $\bar{10}_F$ -plets may actually appear among pentaquarks, depending on the parity of the ground state. The precise number can only be determined after complete anti-symmetrization of the wavefunction, i.e., after full $SU(6)_{FS}$ analysis.

The pentaquark masses must agree with the linear mass formula $M = M_0 + cY$, to the lowest order in the $SU(3)_F$ symmetry breaking, but the M_0 and the c need not be the same for 10_F and $\bar{10}_F$, or even for different 10_F -plets. Indeed, the simplest (“free”) quark model leads to $M_0^{\text{free}}(\bar{10}) = \frac{5}{3}(m_u + m_d + m_s) \simeq 1825$ MeV and $c(\bar{10}) = -\frac{1}{3}(m_s - m_d) \simeq -50$ MeV. Here we used the standard strange and up/down constituent quark masses of 465 MeV and 315 MeV, respectively. These results are to be compared with the “experimental” values of $M_0(\bar{10}) = \frac{1}{3}(M_\Theta + 2M_\Xi) \simeq 1755$ MeV and $c(\bar{10}) = \frac{1}{3}(M_\Theta - M_\Xi) \simeq -107$ MeV, respectively, which are based on the measured masses of Θ_{10}^+ and Ξ_{10}^{--} . The substantial discrepancy in the value of $c(\bar{10})$ should be accounted for by the addition of a hyperfine interaction.

As mentioned above there are four 10_F -plets in the C.G. series, which means e. g. that in the q^4 subsystem there are three (flavour) basis vectors of permutation symmetry $[31]_F$,⁸ and one of symmetry $[4]_F$ (see Appendix A). Each of the three basis vectors of symmetry $[31]_F$ give $M(\Omega) = 2\bar{m} + 3m_s$ and $M(\Delta) = 4\bar{m} + m_s$, where $2\bar{m} = m_u + m_d$. This yields $M_0^{\text{free}}(10) = \frac{1}{3}(M_\Omega + 2M_\Delta) = \frac{5}{3}(m_u + m_d + m_s) \simeq 1825$ MeV. and $c(10) = \frac{1}{3}(M_\Delta - M_\Omega) = -\frac{2}{3}(m_s - m_d) \simeq -100$ MeV, i. e. a twice larger splitting than predicted in the $\bar{10}_F$.

The fourth 10_F -plet mixes with a part of the weight diagram of the 35_F -plet which forms a decimet. After diagonalization the mass matrix is

$$M_\Omega = \begin{pmatrix} M_+(\Omega) & 0 \\ 0 & M_-(\Omega) \end{pmatrix} = \begin{pmatrix} 5m_s & 0 \\ 0 & 2\bar{m} + 3m_s \end{pmatrix}. \quad (1)$$

This shows that the mixing is ideal in the “free” quark model. Similarly for the Δ state mass matrix we get

$$M_\Delta = \begin{pmatrix} M_+(\Delta) & 0 \\ 0 & M_-(\Delta) \end{pmatrix} = \begin{pmatrix} 5\bar{m} & 0 \\ 0 & 4\bar{m} + m_s \end{pmatrix}. \quad (2)$$

There are two possible ways of associating these mixed states into two decimets:

(a) one may group $M_+(\Delta)$ and $M_+(\Omega)$ into one decimet, and $M_-(\Delta)$ and $M_-(\Omega)$ into another. In this case we have $M_0(10_+) = \frac{5}{3}(m_u + m_d + m_s) \simeq 1825$ MeV and $c(10_+) =$

⁸Here $[mn]$ is a partition of 4, or equivalently a Young diagram with four boxes altogether, m of which are in the first row and n in the second one

$-\frac{5}{3}(m_s - \bar{m}) \simeq -250$ MeV for the $(M_+(\Delta), M_+(\Omega))$ pair and $M_0(10_-) = \frac{5}{3}(m_u + m_d + m_s) \simeq 1825$ MeV and $c(10_-) = -\frac{2}{3}(m_s - \bar{m}) \simeq -100$ MeV, for $(M_-(\Delta), M_-(\Omega))$. One can see that $M_0(10_+) = M_0(10_-) = M_0(\bar{10})$, but the corresponding splittings are larger by a factor of 5 and 2 respectively than for the antidecuplet where $c(\bar{10}) = -50$ MeV only.

(b) or one may group $M_+(\Delta)$ and $M_-(\Omega)$ into one decimet, and $M_-(\Delta)$ and $M_+(\Omega)$ into another. In this case we find two “ideally mixed” 10_F -plets, one heavier, with a hidden $s\bar{s}$ pair and $M_0(10_h) = \frac{1}{3}[4(m_u + m_d) + 7m_s] \simeq 1925$ MeV, and $c(10_h) = -\frac{4}{3}(m_s - m_d) \simeq -200$ MeV, and another, lighter one with a hidden light quark pair ($u\bar{u}$, or $d\bar{d}$), with $M_0(10_l) = 2(m_u + m_d) + m_s \simeq 1715$ MeV and $c(10_l) = -(m_s - m_d) \simeq -150$ MeV. The numerical values are based on the usual quark masses (see above).

Hyperfine interactions We shall work with the two most commonly used hyperfine interactions: (1) the colour-spin (CS) model, and (2) the flavour-spin (FS) model. The former is based on the one-gluon exchange (OGE), and used to be the mainstay of hadron spectroscopy for well over 20 years; the latter is based on the pseudoscalar (or Goldstone) boson exchange (GBE), and has been promoted over the past eight years or so [15], in response to certain phenomenological shortcomings of the CS model in the baryon spectra.⁹ One should also remember that the FS model led to the prediction of stable even parity heavy-flavoured pentaquarks [16], presently called Θ_c and Θ_b , long before the recent experimental discoveries of light-flavour pentaquarks.¹⁰ Even more recently it was shown that the FS model can also accommodate the light pentaquark Θ^+ with the (minimal) quark content $uudd\bar{s}$ [18] as well as the whole even parity antidecuplet [19, 20].

One must merely augment the “free” quark model predictions by the hyperfine-induced 10_F , and $\bar{10}_F$ -plet mass shifts $E_{\text{HF}}(10_F)$, $E_{\text{HF}}(\bar{10}_F)$, as follows

$$M_0(10_F) = M_0^{\text{free}}(10_F) + E_{\text{HF}}(10_F) \quad (3)$$

$$M_0(\bar{10}_F) = M_0^{\text{free}}(\bar{10}_F) + E_{\text{HF}}(\bar{10}_F), \quad (4)$$

in order to obtain the observable 10_F - and $\bar{10}_F$ -plet median masses $M_0(10_F)$, $M_0(\bar{10}_F)$. Here $E_{\text{HF}}(10_F)$, $E_{\text{HF}}(\bar{10}_F)$ are the model-dependent hyperfine-induced mass shifts, such as those in Eqs. (6) and (8) below. This is true for states that do not mix. But for states that do mix this is not necessarily correct, as the HFI may change the energies of individual multiplets depending on their flavour content and permutation symmetry. That will not be necessary here for, as we shall see below, the lowest states are always those which do not mix. The reason that the lowest lying states do not mix is that they all originate from q^4 subsystems with $[31]_F$ symmetry.

The Colour-Spin model The schematic¹¹ hyperfine interaction in the CS model is the colour-spin operator

$$V_{cm} = -C_{cm} \sum_{i < j}^5 \lambda_i^C \cdot \lambda_j^C \vec{\sigma}_i \cdot \vec{\sigma}_j. \quad (5)$$

⁹The FS interaction correctly reproduces the level ordering of even and odd parity baryon states both in non-strange and strange sectors while the CS model does not.

¹⁰The first sighting of the charmed pentaquark Θ_c has recently been reported [17].

¹¹Due to our neglect of spatial dependence in the interaction Hamiltonian.

Table 1

The colour-spin hyperfine interaction expectation values for the lowest-lying even and odd parity $J = 1/2$ pentaquarks. Each state is labelled by colour-spin, colour, spin and flavour indices representing the dimensions of $SU(6)_{CS}$, $SU(3)_C$, $SU(2)_S$ and $SU(3)_F$ representations. The \bar{q} defined by $|\bar{6}_{CS}, \bar{3}_C, 2_S, \bar{3}_F\rangle$ is coupled to the q^4 state given in the first column in each case.

q^4 state	$q^4\bar{q}$ state	Parity	$\langle V_{cm} \rangle / C_{cm}$
$ 210_{CS}, 3_C, 1_S, \bar{6}_F\rangle$	$ 70_{CS}, 1_C, 2_S, \bar{10}_F\rangle$	+	-40
$ 210_{CS}, 3_C, 1_S, 15_F\rangle$	$ 70_{CS}, 1_C, 2_S, 10_F\rangle$	+	-40
$ 105_{CS}, 3_C, 1_S, \bar{6}_F\rangle$	$ 70_{CS}, 1_C, 2_S, \bar{10}_F\rangle$	-	-24
$ 105'_{CS}, 3_C, 1_S, 15_F\rangle$	$ 20_{CS}, 1_C, 2_S, 10_F\rangle$	-	-20

where the sum runs over all pairs, the particle 5 being an antiquark, i.e., $\lambda_5 \equiv -\lambda_5^*$. Here λ_i^C are the Gell-Mann matrices for colour $SU(3)_C$, and $\vec{\sigma}_i$ are the Pauli spin matrices. From the fit to the $\Delta - N$ mass splitting $\Delta - N = 16C_{cm} \simeq 300$ MeV one finds $C_{cm} \simeq 18.75$ MeV.¹² The results are exhibited in Table 1 for the four different flavour and parity cases, as derived in Appendix B, where the technical details are relegated to.

From the same Table 1 one can also see that the 10_F -plet is either degenerate with, or is heavier than the $\bar{10}_F$ -plet by

$$E_{\text{HF}}(\bar{10}_F) - E_{\text{HF}}(10_F) = \begin{cases} 0 & + \text{ parity} \\ -4C_{cm} \simeq -75 \text{ MeV} & - \text{ parity} \end{cases} \quad (6)$$

which is the statement we set out to prove.

The Flavour-Spin model In the FS model the qq and $q\bar{q}$ interactions are treated differently. The qq interaction has a flavour-spin structure and in the following we shall employ only its schematic form [15]:

$$V_\chi = -C_\chi \sum_{i < j}^4 \lambda_i^F \cdot \lambda_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j. \quad (7)$$

where the sum runs over qq pairs only. Here λ_i^F are the Gell-Mann matrices for flavour $SU(3)_F$. The constant C_χ has been determined from the Δ -N mass splitting as $C_\chi \simeq 30$ MeV.

The nature of the $q\bar{q}$ interaction is different, however: this interaction was parametrized as a spin-spin one without flavour dependence. Here we shall assume that such a potential holds for any light $q\bar{q}$ pair, and that it does not affect the qq pairs. This brings about the same amount of attraction to every $SU(3)_F$ multiplet and is only a function of the spin of the state. One can reduce our study of the $q^4\bar{q}$ system to the study of the q^4 subsystem (see Appendix C). The results are tabulated in Table 2. From there one can see that the

¹²The hyperfine Hamiltonian Eq. (5) expectation values in the $q^4\bar{q}$ system can be calculated by way of a formula in [21, 22].

Table 2

The flavour-spin hyperfine interaction expectation values for even and odd parity $J = 1/2$ pentaquarks. Column 2 indicates the $SU(3)$ flavour multiplet resulting from the coupling of $|\psi_i^+\rangle$ and $|\psi_i^-\rangle$ to \bar{q} , see Appendix C.

q^4 state	$q^4\bar{q}$ state	$\langle V_\chi \rangle / C_\chi$
$ \psi_1^+\rangle$	$\overline{10}_F$	- 28
$ \psi_2^+\rangle$	10_F	- 64/3
$ \psi_1^-\rangle$	$\overline{10}_F$	- 28/3
$ \psi_2^-\rangle$	10_F	- 8

mean mass of the 10_F -plet is heavier than the $\overline{10}_F$ -plet's one

$$E_{\text{HF}}(\overline{10}_F) - E_{\text{HF}}(10_F) = \begin{cases} -\frac{20}{3}C_\chi \approx -200 \text{ MeV} & + \text{ parity} \\ -\frac{4}{3}C_\chi \approx -40 \text{ MeV} & - \text{ parity} \end{cases} \quad (8)$$

unless an additional spin-spin interaction acting only on $q\bar{q}$ pairs is introduced, as in Ref. [18], in which case an exact 10_F - $\overline{10}_F$ degeneracy could be recovered.

Comparison with experiment and conclusions Based on the above results in the CS and FS models, we can make predictions for the masses of the decuplet relative to the antidecuplet. We rely on the fact that $M_0^{\text{free}}(10_F) = M_0^{\text{free}}(\overline{10}_F)$ for the lowest states and that the splitting in the decuplet are expected to be larger than in the antidecuplet. The hyperfine interaction changes M_0^{free} according to Eqs. (3), (4),(6) and (8). This means for example that for even parity states the CS model predicts the same median mass both for the decuplet and the antidecuplet but in the FS model the antidecuplet median mass should be much lower. Smaller differences appear for odd parity pentaquarks median mass in both models. Our results can be summarized in the form of mass relations (or “sum rules”) as follows

$$M_\Theta + 2M_\Xi = M_\Omega + 2M_\Delta + 3 \left[E_{\text{HF}}(\overline{10}_F) - E_{\text{HF}}(10_F) \right] \quad (9)$$

and

$$M_\Delta - M_\Omega = \alpha(M_\Theta - M_\Xi), \quad (10)$$

where α can be $= 2, 3, 4, 5$, depending on the decimet in question. We recall that the lowest-lying decimet corresponds to $\alpha = 2$. This mass formula could be substantially altered by the $SU(3)_F$ symmetry breaking in the hyperfine interaction, which we have not taken into account. Now let us look into the Particle Data Group's (PDG) tables [24] to see whether or not we can find possible candidates for the pentaquark decuplet. All of the presently known excited Ω^- states, with as yet undetermined spins and parities, lie above 2200 MeV, (e.g. $\Omega^-(2250)$). Of all the low-lying Ξ hyperons, only two ($\Xi(1690)$ and $\Xi(1950)$) have spins that might (albeit need not) be consistent with our $J^P = 1/2^+$

requirement. Similarly, only two observed Σ hyperons are allowed by this spin-parity assumption, *viz.* $\Sigma(1660)$, $\Sigma(2250)$. We may instantly eliminate the $\Sigma(2250)$ and $\Xi(1690)$ candidates, as too heavy and too light, respectively. The only possible “triplet” of states ($\Sigma(1660)$, $\Xi(1950)$, $\Omega(2250)$) satisfies the $(10)_F$ -plet mass “equidistance rule” $M_\Xi - M_\Sigma = M_\Omega - M_\Xi$, but with an SU(3) symmetry breaking mass splitting ($c_{10} = -300$ MeV) that is roughly three times bigger than the $\overline{10}_F$ -plet one ($c_{\overline{10}} = -107$ MeV). But then we remember option (b) above, in which the parameter α in the sum rule Eq. (10) can take on the value of three, which is not inconsistent with the present experimental value. It is, however, well known that there are no $J^P = 1/2^+$ Δ states around 1360 MeV. Finally, this decimet’s mean mass $M_0(10_F) = 1660$ MeV, together with $M_0(\overline{10}_F) = 1755$ MeV, disagrees even with the sign of Eq. 8, for both parities.

Identification of non-exotic pentaquarks is an empirical question that will have to be settled by experiment. If the pentaquarks turn out to be compact hadronic states, then the question of the failure to find experimentally pentaquarks that satisfy the predicted mass relations will have to be addressed. At this stage one can only say that the predicted mass relations should not have been a surprise: Both of these hyperfine interactions are of the two-body kind, thus being proportional to (at most) the quadratic Casimir operator(s) of SU(3) and/or SU(6), which do not distinguish between a representation (multiplet) and its conjugate. Consequently any multiplet is likely to be degenerate with its own conjugate, subject, of course, to the afore discussed caveats.

In other words, neither of the two hyperfine interactions is taking full advantage of the postulated symmetries, i.e., of the allowed SU(3) and/or SU(6) group theoretical structures [25]. Thus, the (two-body) CS model certainly cannot mimic the full QCD’s SU(3) and/or SU(6) algebraic structure (irrespective of any spatial or temporal dependence), and the (two-body) FS model does not include all possible phenomenological interactions. One remedy to these problems that obviously suggests itself is to include three-body forces of either the CS or the FS kind. This remains a task for the future.

Acknowledgments

One of us [V.D.] wishes to thank Mr. Predrag Krstonošić for his help with the figure and the staff of Fundamental Theoretical Physics Group, Université de Liège, for their kind hospitality.

A. Appendix: The flavour SU(3) “wave functions” of the decuplet

Here we give the expressions of the flavour wave functions used to derive the decimet states’ masses in the “free” quark model and the mass matrices (1) and (2). In writing the explicit form of the flavour part of the decuplet wave functions, first we construct the basis vectors of the q^4 subsystem. For this purpose it is convenient to use the Young-Yamanouchi-Rutherford basis [26] which allows one to specify the permutation symmetry of the last two particles, here 3 and 4. Then we couple the antiquark to the q^4 subsystem with the help of SU(3) Clebsch-Gordan coefficients. Below we give the explicit form for the flavour part of four decuplet pentaquark Ω .

1) The state $[31]_F$ is symmetric under the permutation of particles 1 and 2 (the so called transposition (12)), but antisymmetric under the permutation of particles 3 and 4 (trans-

position (34))

$$|\Omega_A^-\rangle = -\frac{1}{2}|ss[u, s]\bar{u} + ss[d, s]\bar{d}\rangle \quad (11)$$

2) The state $[31]_F$ symmetric under both transpositions (12) and (34)

$$|\Omega_S^-\rangle = -\frac{1}{2\sqrt{2}}|(ss\{u, s\} - \{u, s\}ss)\bar{u} + (ss\{d, s\} - \{d, s\}ss)\bar{d}\rangle \quad (12)$$

3) The state $[31]_F$ antisymmetric under the (12), but symmetric under the (34) transposition

$$|\Omega_3^-\rangle = \frac{1}{2}|[u, s]ss\bar{u} + [d, s]ss\bar{d}\rangle \quad (13)$$

4) The totally symmetric state $[4]_F$

$$|\Omega_4^-\rangle = -\frac{1}{2\sqrt{6}}|(ss\{u, s\} + \{u, s\}ss)\bar{u} + (ss\{d, s\} + \{d, s\}ss)\bar{d} - 4ssss\bar{s}\rangle. \quad (14)$$

The normal order 1234 for quarks holds everywhere. We use the notation $[a, b] = ab - ba$ and $\{a, b\} = ab + ba$, the Young-Yamanouchi phase convention for the permutation group, and the SU(3) CG coefficients as well as the SU(3) phase conventions of Refs. [27, 28].

Now recall that both the 10_F - and the 35_F -plet appear in the CG series $15 \times \bar{3} = 10 + 35$ or equivalently $[4] \times [11] = [311] + [51]$. This means that there is a subset of states of the 35_F -plet which have the same overall quantum numbers Y, I, I_3 as those in a decuplet. In particular a fifth Ω^- state, denoted here by $|\Omega_5^-\rangle$ can be obtained:

$$|\Omega_5^-\rangle = \frac{1}{\sqrt{12}}|(ss\{u, s\} + \{u, s\}ss)\bar{u} + (ss\{d, s\} + \{d, s\}ss)\bar{d} + 2ssss\bar{s}\rangle \quad (15)$$

It is precisely this state that mixes with $|\Omega_4^-\rangle$. This mixing gives rise to the mass matrix (1). The flavour basis vectors for the pentaquark Δ^- are obtained by the replacement $s \leftrightarrow d$ and $\bar{s} \leftrightarrow \bar{d}$ above and the corresponding mixing gives rise to Eq. (2).

There are other ways of writing the basis states, but the advantage of writing them in the above form is that they can then be naturally coupled to other parts of the wave functions depending on spin, colour or space in order to properly satisfy the Pauli principle for the subsystem of quarks.

B. Appendix: The colour-spin model

We start by using SU(6) representations for the pentaquarks. In the OGE model we consider the direct products $SU(2)_S \times SU(3)_C$, as subgroups of SU(6). Then the q^3 and the $q\bar{q}$ subsystems are described by the 56 and the 35 irreps of SU(6) and for the $q^4\bar{q}$ system we get the SU(6) Clebsch-Gordan series

$$56 \times 35 = 56 + 70 + 700 + 1134 \quad (16)$$

First we have to consider the compatibility of the symmetry of the flavour part of the q^4 subsystem with either 10_F - or $\bar{10}_F$ -plets of the $q^4\bar{q}$ system. We have the following four

cases:

a) $J^P = 1/2^+$ pentaquarks belonging to the $\overline{10}_F$ -plet.

The $SU(3)_F$ symmetry of the q^4 subsystem is $[22]_F$ in order to give rise to the $\overline{10}_F$ -plet because $[22]_F \times [11]_F = \overline{10}_F + 8_F$, where $[f]$ represents a given Young tableau of partition $[f]$ and O,C,F and S stand for the orbital, colour, flavour and spin degrees of freedom, respectively. Note that from this decomposition of the direct product it follows that $\overline{10}$ and 8 are degenerate if $SU(3)_F$ is an exact symmetry. The lowest state of even parity is $[31]_O$, containing one quark in the p -shell. This gives $[31]_O \times [22]_F = [31]_{OF} + [211]_{OF}$. The $[211]_{OF}$ part is combined with its conjugate $[31]_{CS}$ in order to obtain a totally antisymmetric q^4 state. Here $[31]_{CS}$ has dimension 210 in $SU(6)$ and it is obtained from the inner product $[31]_{CS} = [211]_C \times [22]_S$ which means that the q^4 subsystem has spin zero, so that the total spin is $S = S_{\bar{q}} = 1/2$. The total angular momentum is therefore $J = 1/2$ or $3/2$ and the interaction Eq. (5) cannot distinguish between them. In $SU(6)$ notation the coupling to the antiquark described by the $\bar{6}$ representation gives $\bar{6} \times 210 = 1134 + 56 + 70$, compatible with the relation (16). The most favourable symmetry multiplet is 70 as implied by the formula given in Ref. [21] or [22]. This state has also been considered in Ref. [29] and is the lowest even parity pentaquark state in the CS model.

b) $J^P = 1/2^+$ pentaquarks belonging to the 10_F -plet.

The $SU(3)_F$ symmetry of the q^4 subsystem is $[31]_F$ which gives rise to $15_F \times \bar{3}_F = 10_F + 8_F + 27_F$, all these multiplets being degenerate if the $SU(3)_F$ symmetry is exact. In the orbital and the flavour space the direct product is $[31]_O \times [31]_F = [4]_{OF} + [31]_{OF} + [22]_{OF} + [211]_{OF}$. From here we choose $[211]_{OF}$ which combined with $[31]_{CS}$ gives the most favourable q^4 totally antisymmetric state. This implies that the $SU(6)$ direct product obtained from the coupling to the antiquark is the same as for the case a) and the most favourable multiplet is again 70.

c) $J^P = 1/2^-$ pentaquarks belonging to the $\overline{10}_F$ -plet.

The symmetry of the q^4 subsystem in the $SU(3)_F$ space is $[22]_F$, like in the case a). The lowest state of odd parity is $[4]_O$ so one gets $[4]_O \times [22]_F = [22]_{OF}$. In order to obtain a totally antisymmetric wave function for q^4 one must combine the $[22]_{OF}$ part with its conjugate in the CS space, the $[22]_{CS} = [211]_C \times [22]_S$ state, of dimension 105 in $SU(6)$. Then the coupling of the antiquark gives $\bar{6} \times 105 = 560 + 70$. The multiplet with the most favourable symmetry is 70. This state appears as state V in Ref. [22] and is the lowest odd parity pentaquark state in the CS model.

d) $J^P = 1/2^-$ pentaquarks belonging to the 10_F -plet.

The $SU(3)_F$ symmetry of the q^4 subsystem is $[31]_F$ as in case b). The lowest state of odd parity is $[4]_O$ so one gets $[4]_O \times [31]_F = [31]_{OF}$. Then the Pauli principle requires to combine it with $[211]_{CS}$. The coupling to the antiquark gives $\bar{6} \times 105' = 540 + 70 + 20$. The most favourable multiplet is 20.

Table 1 lists all the above $q^4\bar{q}$ states together with the states of the corresponding q^4 subsystem. Note that the $SU(6)$ representations associated to $q^4\bar{q}$ and q^4 are, in linear

combinations, consistent with the considerations made in Ref. [30]. This indicates that the even parity antidecuplet and decuplet states are degenerate. The odd parity ones are not. Moreover even though the contribution of the CS hyperfine attraction is roughly two times larger for even parity states than for odd parity ones, the extra unit of orbital excitation $\hbar\omega \simeq 500$ MeV [15], carried by the even parity state leads to

$$E^+ - E^- = \begin{cases} \frac{1}{2}\hbar\omega - 16C_{cm} \approx -50 \text{ MeV} & \text{antidecuplet} \\ \frac{1}{2}\hbar\omega - \frac{40}{3}C_{cm} \approx 0 & \text{decuplet} \end{cases} \quad (17)$$

with $C_{cm} \simeq 18.75$ MeV. This schematic estimate implies that in the antidecuplet case the $1/2^+$ state is expected somewhat below the $1/2^-$ state, in agreement with Ref. [31], whereas in the decuplet channel they would be practically degenerate.

C. Appendix: The flavour-spin model

The overall parity is determined by that of the q^4 subsystem. The available SU(6) representations describing the q^4 subsystem are given by the direct product decomposition

$$6 \times 6 \times 6 \times 6 = 126 + 3(210) + 2(105) + 3(105') + 15 \quad (18)$$

In the GBE model the lowest totally antisymmetric $J^P = 1/2^+$ states constructed in the FS coupling scheme are given by

$$|\psi_1^+\rangle = |[31]_O [211]_C [1^4]_{OC} ; [22]_F [22]_S [4]_{FS}\rangle \quad (19)$$

$$|\psi_2^+\rangle = |[31]_O [211]_C [1^4]_{OC} ; [31]_F [31]_S [4]_{FS}\rangle \quad (20)$$

In each case the colour part is uniquely defined. It gives rise to a totally antisymmetric OC state if combined with $[31]_O$ which contains one p -shell quark state. Together with the parity of the antiquark this leads to $L=1$ even parity states. As the FS part is totally symmetric one obtains totally antisymmetric q^4 states. These states were for the first time considered in Ref. [16] in the context of even parity heavy pentaquarks, presently denoted in the literature by Θ_c and Θ_b . These were also the two states used in Ref. [18]. The first has spin $S = 0$ and the second $S = 1$. The coupling to the antiquark spin leads to a total $S = 1/2$ for both and the coupling to $L = 1$ gives $J = 1/2$ or $3/2$.

The SU(6) flavour-spin state $[4]_{FS}$ is totally symmetric i. e. it belongs to the representation (126) of SU(6). In this situation the only possible SU(6) representations of a $q^4\bar{q}$ system are given by

$$126 \times \bar{6} = 700 + 56 \quad (21)$$

It follows that the above states are compatible only with the (700) SU(6) representation which contains both $\bar{10}_F$ and 10_F having $J = 1/2$. The state $|\psi_1^+\rangle$ corresponds to $\bar{10}_F$ and $|\psi_2^+\rangle$ to 10_F . As Table 2 indicates these states are not degenerate. An additional spin-spin interaction, such as the one considered in Ref. [18], with an adequate strength could however make these two states degenerate.

By analogy $J^P = 1/2^-$ states can be constructed as

$$|\psi_1^-\rangle = |[4]_O [211]_C [211]_{OC} ; [22]_F [31]_S [31]_{FS}\rangle. \quad (22)$$

and

$$|\psi_2^-\rangle = |[4]_O [211]_C [211]_{OC} ; [31]_F [22]_S [31]_{FS}\rangle. \quad (23)$$

where $[31]_{FS}$ is the SU(6) representation (210) which leads to

$$210 \times \bar{6} = 1134 + 56 + 70 \quad (24)$$

in the $q^4\bar{q}$ system. The state $|\psi_1^-\rangle$ is compatible with the representation (1134), the only one which contains $\bar{10}_F$, and $|\psi_2^-\rangle$ is compatible with (1134), (56) or (70)-plet. From Table 2 one can see that these states are also not degenerate. These states have been considered in Ref. [32] in the context of odd parity heavy pentaquarks containing c or b antiquarks.

One can see that for both 10_F - and $\bar{10}_F$ -plets, the even parity state lies far below the odd parity one. Taking into account that the even parity states contain one unit of orbital excitation $\hbar\omega \simeq 500$ MeV [15] and using Table 2 with $C_\chi \simeq 30$ MeV one obtains

$$E(\psi^+) - E(\psi^-) = \begin{cases} \frac{1}{2}\hbar\omega - \frac{56}{3}C_\chi \approx -310 \text{ MeV} & \text{antidecuplet} \\ \frac{1}{2}\hbar\omega - \frac{40}{3}C_\chi \approx -150 \text{ MeV} & \text{decuplet} \end{cases} \quad (25)$$

Thus, for odd parity pentaquarks, both the antidecuplet and the decuplet are expected to be far above the threshold and highly unstable.

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